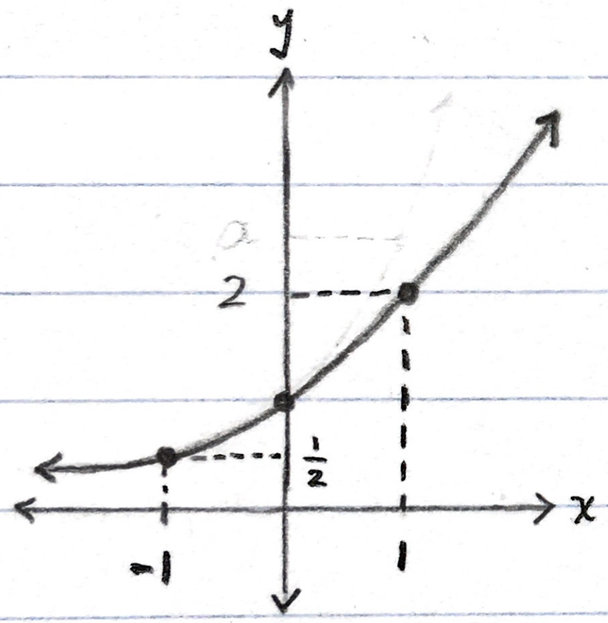
5.3 - The Exponential Function

Def. $\boxed{f(x) = a^x}$; $x \in \mathbb{R}$, $a > 0$, $a \neq 1$ (a is called the <u>base</u>*)

2 cases

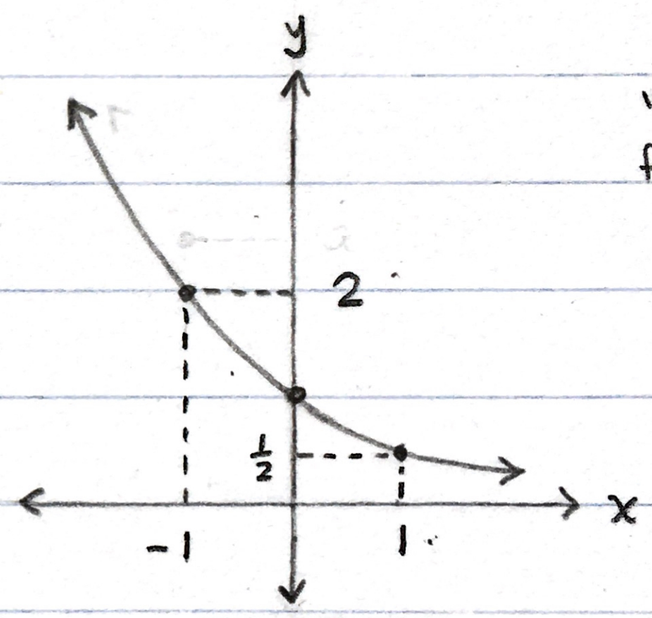
$a > 1$ | $0 < a < 1$

expo "growth" function



y	x	y
$\frac{1}{2}$	-1	2
1	0	1
2	1	$\frac{1}{2}$

← HA: $y = 0$ →



expo "decay" function

$f(x) = 2^x$ ← EXAMPLES → $f(x) = \left(\frac{1}{2}\right)^x$ (aka 2^{-x})

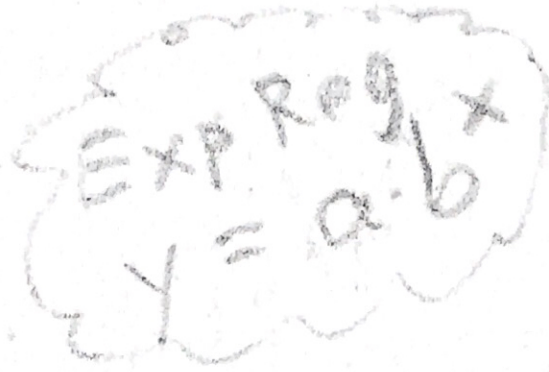
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* Noteworthy base (e): $f(x) = e^x$ ("natural" expo function)

<u>Definition of e</u>: e is the HA of $f(x) = \left(1 + \frac{1}{x}\right)^x$.
(Euler's number)

FYI: $e = \sum_{n=0}^{\infty} \frac{1}{n!}$

5.3 (cont'd) - Key Concepts



I. Modeling $f(x) = Ca^x$, given:

a) data table (ex. (15))

b) 2 points (ex. (31) (33))

c) graph (ex. (67) (69))

II. Growth Application: Compound Interest

a) periodic: $A_p(t) = P(1 + \frac{r}{n})^{nt}$ (ex. (95))

b) continuous: $A_c(t) = Pe^{rt}$ (ex. (97))

III. Decay Applications

a) Half-Life: $A(t) = C(\frac{1}{2})^{t/k}$ (ex. (125))

b) Cooling an Object (5.4 - (141))

---

(15)

x	-3	-2	-1	0	1
y	64	32	16	8	4

(31) $f(0) = 10$ and $f(1) = 20$

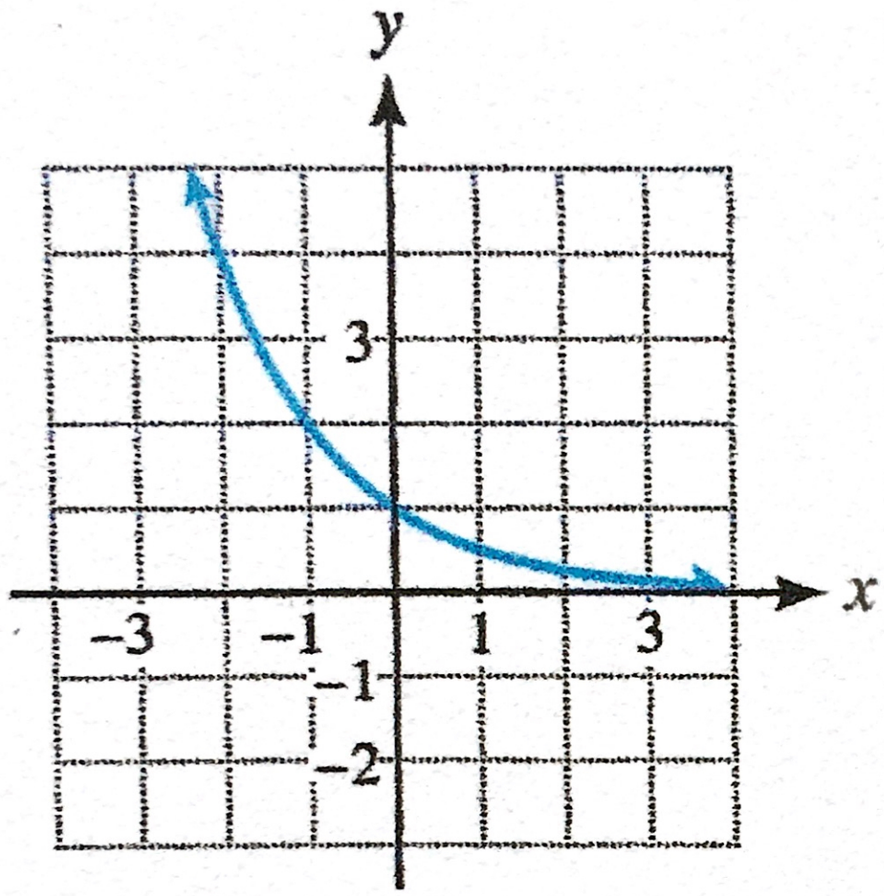
(33) $f(1) = 9$ and $f(2) = 27$

(95) $950 at 3% compounded daily for 20 years.

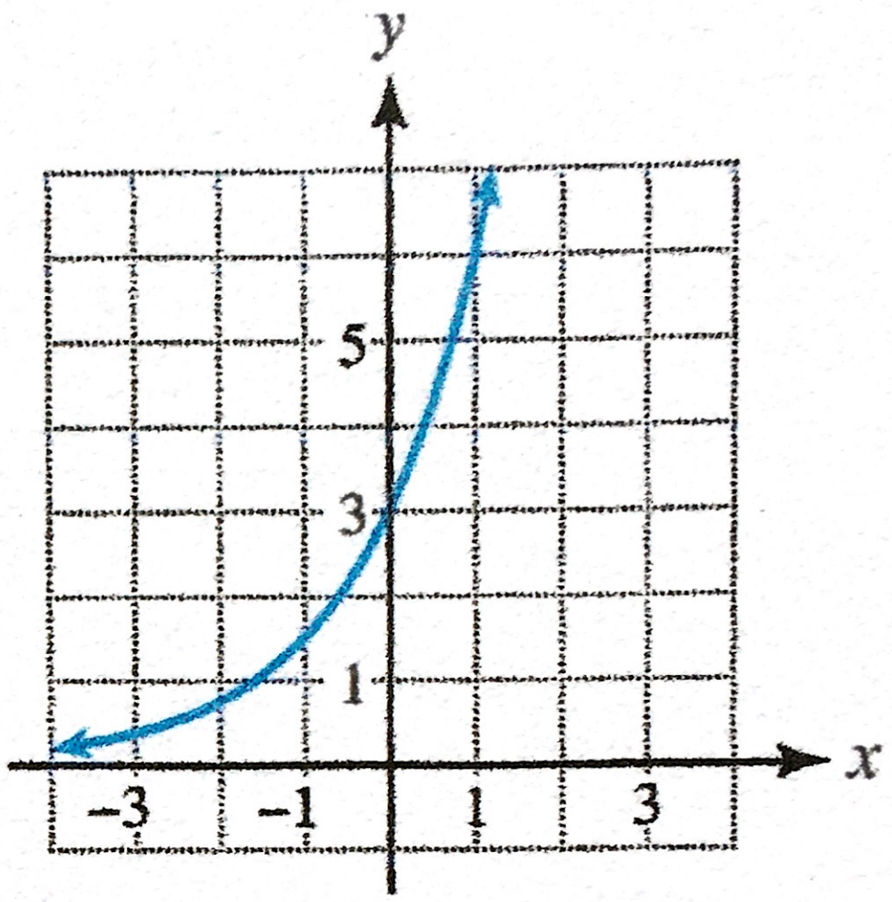
(97) $2,000 at 10% compounded continuously for 8 years.

Exercises 67–70: Use the graph of $y = Ca^x$ to determine values for C and a.

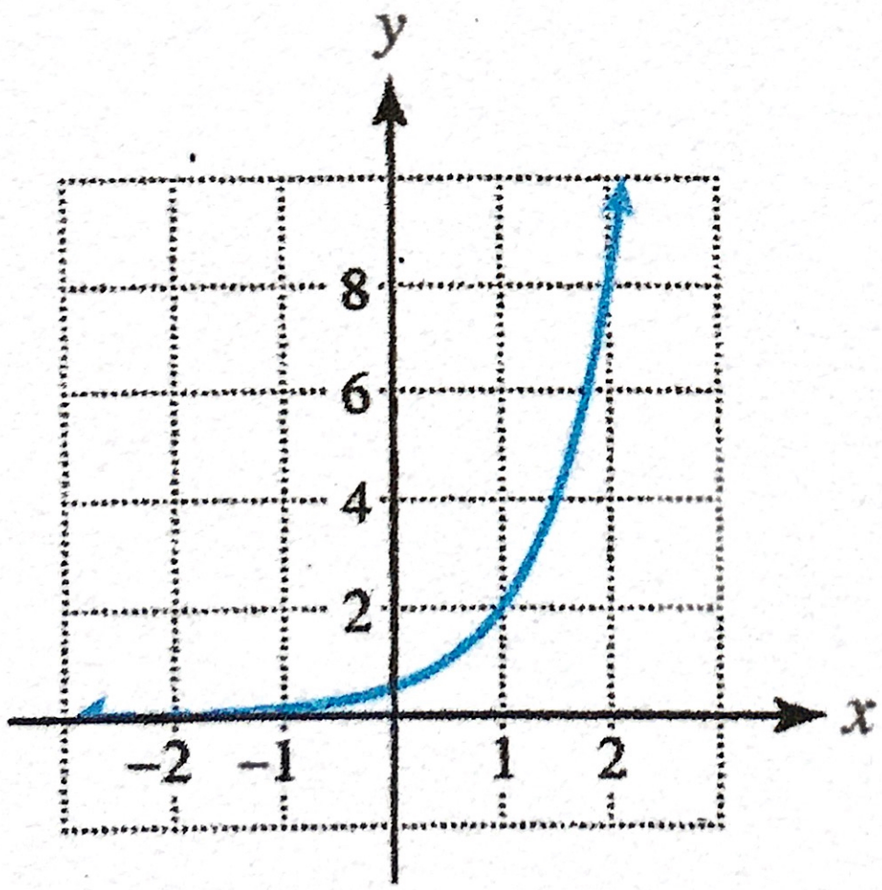
67.



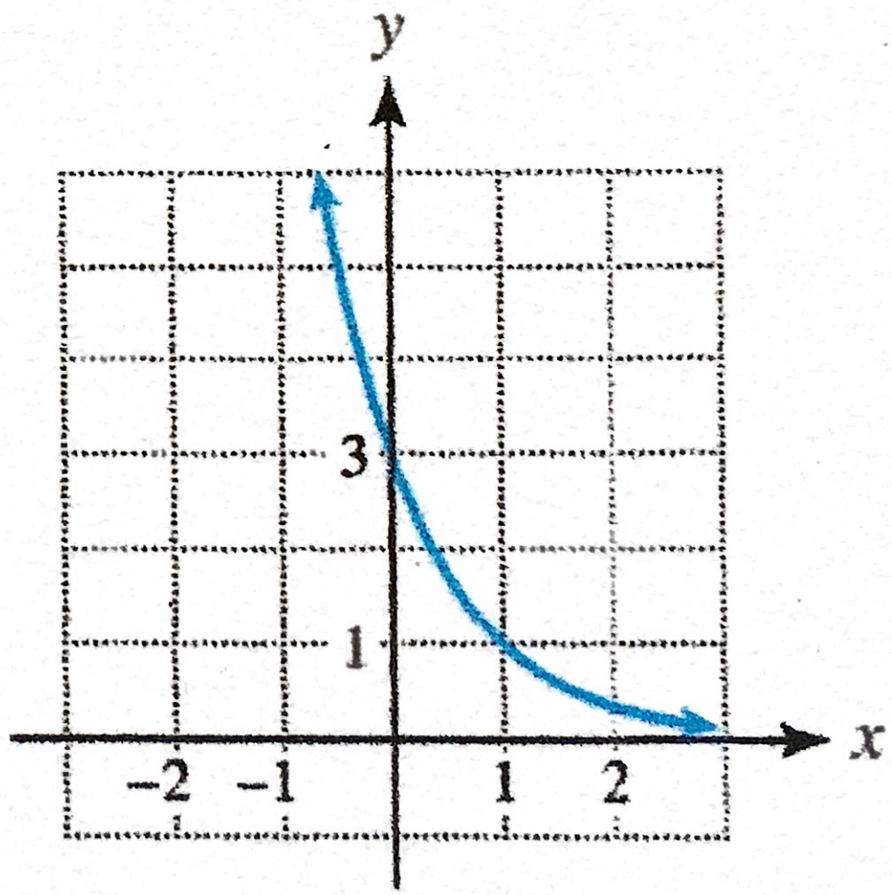
68.



69.



70.



5.3 (cont'd) - Half-Life

Def. If a quantity initially equals C and has a half-life of k, then the amount A remaining after time t is: $\boxed{A(t) = C\left(\frac{1}{2}\right)^{t/k}}$ $\left(\begin{matrix} A(0) = C \\ A(k) = \frac{1}{2}C \end{matrix}\right)$

Example. (125) Half-Life for Twitter. (k)

The half-life for a link on Twitter is 2.8 hours.

(a) Find an exponential function T that gives the percentage of engagements remaining on a typical Twitter link after t hours.

(b) Estimate this percentage after 5.5 hours.

Solution.

t	T
0	100
2.8	50

t-hours

T-%

(a) $T(t) = 100\left(\frac{1}{2}\right)^{t/2.8}$

(b) $T(5.5) \approx 25.6\%$

5.4 - (141) Cooling an Object.

A pot of boiling water with a temp. of 100°C is set in a room with a temp. of 20°C.
The temperature T of the water after x hours is given by: $T(x) = 20 + 80e^{-x}$.

(a) Estimate the temp. of the water after 1 hour.
Find $T(1)$.

(b) How long did it take the water to cool to 60°C?
Find x: $T(x) = 60$.

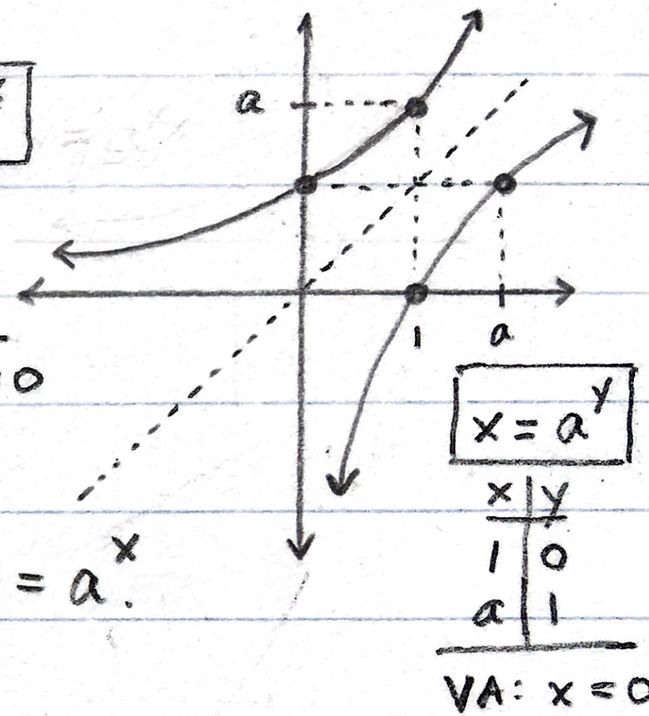
5.4 – The Logarithmic Function

Observations. 1. $y = a^x$ is 1-1.
2. $x = a^y$ is its inverse.

$\boxed{y = a^x}$

x	y
0	1
1	a

HA: $y = 0$



$\boxed{x = a^y}$

x	y
1	0
a	1

Def. Let $\boxed{y = \log_a x}$ denote $x = a^y$.
Then $f^{-1}(x) = \log_a x$ is the inverse of $f(x) = a^x$.

Special Cases of a (e, 10)

Defs. ① $\log x = \log_{10} x$ (base 10 log)
② $\ln x = \log_e x$ (natural log)

} Standard bases in your calculator

Summary.

EXPO	INVERSE
$y = e^x$	$y = \ln x$
$y = 10^x$	$y = \log x$

- $\log_a 1 = 0$ and $\log_a a = 1$
- $\log_a a^x = a^{\log_a x} = x$
- domain = $(0, \infty)$ or $\{x \mid x > 0\}$

5.4 (cont'd) – Key Concepts & Drills

I. Evaluating / Simplifying (w/o calculator)

(•) $7^{\log_7 5}$ (25) $7^{\log_7 2x}$ (use $a^{\log_a x} = x$)

(35) $\log_2 64$ (•) $\log_2 64^x$ (use $\log_a a^x = x$)

II. Finding domains of transformations.

(13) $f(x) = \log(x+3)$ (14) $f(x) = \log(2x-4)$.

(15) $f(x) = \log_2(x^2-1)$

III. Approximating logs with a calculator.

$\log 75$; $\ln 2$

IV. Change-of-base Formula: $\log_a x = \dfrac{\log_b x}{\log_b a}$

Estimate: $\log_2 5$

V. Solving Expo & Log Equations.

(see prelims) (63) $4^x = \frac{1}{16}$; $e^x = 2$; $4^x = 125$

(79) $2^x + 1 = 15$

(87) $\log_2 x = 6$; $\log_3 x = -2$; $\ln x = 2$

(95) $2\log 5x = 4$ (99) $5\ln x - 1 = 6$

VI. Application: Barometric Pressure

VII. Graphing: $f(x) = \log_2(x-2) + 1$

Section 5.4

V.
(prelims)

Conversions: Expo ↔ Log
(extra prep for drill V)

(57) $7^{4x} = 4$ (to log)

(59) $\log_8 x = 3$ (to expo)

VII.
(identity points)

$y = \log_2(x-2) + 1$

x	x-2	y	y+1
3	1	0	1
4	2	1	2
2½	–	–	0

identities

5.4 (cont'd) - VI. Application: Barometric Pressure

Hurricanes are some of the largest storms on Earth.
They are very low pressure areas with diameters of over 500 miles.

The barometric air pressure (in inches of mercury at a distance of x miles from the eye of a severe hurricane) is modeled by the formula:

$$f(x) = 0.48 \ln(x+1) + 27 \quad \text{in Hg}$$

(a) Evaluate $f(0)$ and $f(100)$. Interpret the results.

(c) At what distance from the eye of the hurricane is the air pressure 28 inches of mercury?